Technical Notes

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Fuel Evaporation Rate in Intense Recirculation Zones

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Introduction

COMBUSTOR analysis of high intensity combustors, such as are used in industrial boilers, automotive engines, and jet engines, usually employ one or more stirred reactor zones[1,2] to model the distinct regions in the combustor which have intensely recirculating flow. For example, the flow in the primary region of a jet engine is well represented[3] by a combination of two stirred reactors linked by plug flow of combustion products from the primary stirred zone to the swirler zone. The intense turbulence in a stirred reactor ensures virtually complete mixing of combustion products with reactants as is usually modeled as a chemical-kinetically limited system in which composition and temperature are uniform everywhere. This is an excellent assumption for gaseous species but does not apply to liquid (or solid) fuel droplets (particles). When droplets are injected into a stirred reactor, evaporation occurs and individual droplets decrease in size according to the well-known "d^2-law."[4] In a stirred reactor, the mean liquid fuel concentration will be determined by the residence time distribution function. For average reactor residence times in excess of that required for complete evaporation of a single drop, there will still be liquid fuel present because of drops which have only spent a fraction of this time in the reactor. The mean liquid fuel concentration in a stirred reactor is thus not given by direct application of the d^2-law. Thus, the Sauter Mean Diameter (SMD) is not the characteristic one for fuel droplets in stirred reaction zones. A simple formula is derived and some numerical results are plotted graphically to illustrate the different mean evaporation rate of liquid fuel in an ambient and highly stirred environment.

Analysis

The rate of evaporation of single droplets is given by

$$d\alpha/dt = -1.5\alpha^o \lambda d/d_o^3 \tag{1}$$

where $d\alpha/dt$ is the evaporation rate, α^o is the initial mass fraction of liquid fuel, λ is an evaporation rate constant, d is the drop diameter at t, and d_o is the initial drop diameter.

This equation is obtained using the empirical d^2-law[4]

$$d^2 = d_o^2 - \lambda t \tag{2}$$

Substituting Eq. (2) in Eq. (1) and integrating, one obtains

$$\frac{\alpha_t}{\alpha_o} = \left\{\frac{d}{d_o}\right\}^3 = \left\{1 - \frac{\lambda t}{d_o^2}\right\}^{3/2} \tag{3}$$

This result also agrees with physical intuition.

Inspection of Eq. (3) shows that for times $t > d_o^2/\lambda$ all liquid fuel has evaporated. It is assumed that the droplets are small compared with the volume of the reactor and are uniformly dispersed through it, but remain independent throughout their residence time in the reactor. Droplets entering the stirred reactor which have the same residence time, will have the same history and will have evaporated to the same extent when leaving the reactor. The liquid fuel fraction at the exit, equal to the mean value in the reactor, is then given by summing up all fractions of fuel having different residence times.

$$\alpha = \frac{1}{\tau}\int_o^\infty \alpha_t e^{-t/\tau}\, dt \tag{4}$$

where τ is the mean residence time in the reactor and $(1/\tau)e^{-t/\tau}$ is the residence time distribution function.

Equation (4) cannot be integrated as it stands, since α_t goes to zero as t goes to $d_o^2/\lambda t$. Equation (4) is only meaningful in this range, thus the integration limits can be changed:

$$\alpha = \frac{1}{\tau}\int_o^{d_o^2/\lambda t} \alpha_t e^{-t/\tau}\, dt \tag{5}$$

Integrating Eq. (5), one obtains

$$\frac{\alpha}{\alpha_o} = 2e^{-d_1^2/\lambda t}\left[\sum_{n=o}^{\infty}\left\{\frac{d_o^2}{\lambda\tau}\right\}^{n+1}\frac{1}{(5+2n)n!}\right] \tag{6}$$

The value of λ is obtained from the following expression[1]:

$$\lambda = \lambda^o(1.0 + 0.276 Re^{1/2} Sc^{1/3})$$

$$\lambda^o = (8k/\rho_1 C_p)\ln[1 + C_p(T - T_1)/L]$$

The infinite sum in Eq. (6) converges rapidly except for large values of $d_o^2/\lambda\tau$. It is possible to have large values of $d_o^2/\lambda\tau$ when the reactor is near blowout ($T < 800°$K). A useful alternate form for α/α^o is given by the following[5]:

Let $y = d_o^2/\lambda\tau$ and $f = 1$. Therefore

$$(\alpha/\alpha^o) = 2ye^{-y}\sum_{n=o}^{\infty}\{y^n/n!(2n+1)\}$$

$$= 2ye^{-y}\int_o^1 t^2 e^{yt}/(t)^{1/2}\, dt$$

where

$$1/(2n+5) = \int_o^1 x^{2n+4}\, dx$$

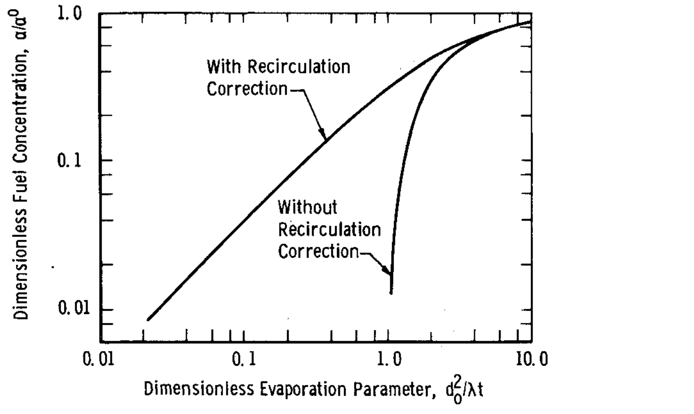

Fig. 1 Effect of recirculation on fuel evaporation in an intense recirculating region.

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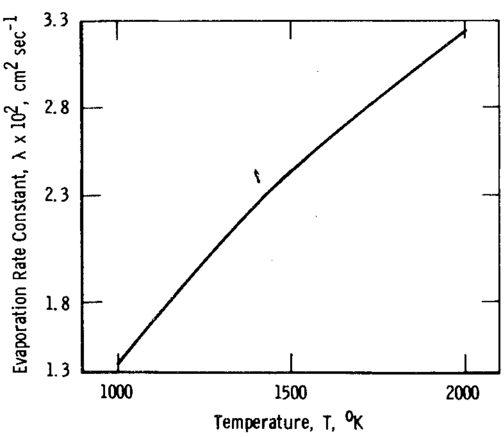

Fig. 2 Evaporation rate constant for jet-fuel vs mean reactor temperature (data from Ref. 1).

Using standard Laplace Transform tables, one obtains

$$(\alpha/\alpha^o) = y e^{-y}(d^2/dy^2)\{(\pi/-y)^{1/2}\,\mathrm{erf}(-y)^{1/2}\} = (e^{-y}/y)[\tfrac{3}{4}(\pi/-y)^{1/2}\,\mathrm{erf}(-y)^{1/2} - e^{+y}(\tfrac{3}{2}-y)]$$

From Ref. 6,

$$w(z) = e^{-z^2}\,\mathrm{erf}\,c(-iz) = e^{-z^2}[1-\mathrm{erf}(-iz)]$$

$$\mathrm{erf}(-y)^{1/2} = \mathscr{I}_m\{e^y[1-w(-y)^{1/2}]\} = i e^y \mathscr{I}_m[w(y)^{1/2}]$$

Therefore

$$\alpha/\alpha^o = (1/y)\{\tfrac{3}{4}(\pi/y)^{1/2}\mathscr{I}_m[w(y)^{1/2}] - (\tfrac{3}{2}-y)\}$$

Note: $\mathscr{I}_m[w(z)] = 2/(\pi)^{1/2}$ times Dawson's Integral.

Results and Discussion

Equation (6) was evaluated for a range of values of $d_o^2/\lambda\tau$ and the results are plotted in Fig. 1. Equation (3) was also evaluated and the results plotted in Fig. 1 to indicate the difference in fuel evaporation rate when recirculation is taken into account. It is hoped that use of Eq. (6) instead of Eq. (3) in stirred reactor calculations may improve pollutant predictions. Finally, values of λ, the evaporation rate constant, were evaluated and plotted in Fig. 2 as a function of reactor temperature T. Data for the convection correction to λ were obtained from Ref. 1.

Conclusions

An expression has been developed for the mean liquid fuel concentration in an intensely recirculating flow region. The expression allows liquid fuel evaporation rates to be evaluated with finite fuel concentrations predicted even for residence times greater than the critical value for single drops. The mean fuel evaporation rate may thus be significantly smaller than that for single drops. In a jet engine combustor in which strong recirculation is generated the fuel will not be fully evaporated.

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Analytical Approximate Calculation of Optimal Low-Thrust Energy Increase Trajectories

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Introduction

An approximate analytic solution to optimal low thrust constant acceleration energy increase trajectories is presented. The spacecraft to be considered shall ascend from a circular orbit around the spherical central body to some specified energy level in minimal time. Analytical solutions to this problem have previously been found by Lawden,[1] whose solution does not predict the oscillatory character of the optimal control program. The solution by Breakwell and Rauch[2] holds an accuracy of about 1% over a small number of revolutions around the central body only. Jacobson and Powers[3] developed an accurate small parameter perturbation solution with a correct prediction of the oscillatory nature of the optimal control program. They all used the classical Newtonian formulation of the equations of motion.

The solution presented in this Note is based on a two-variable asymptotic expansion[4] of the equations of motion and the Lagrange multiplier equations. The regularized form[5] of the equations of motion leads to a very high accuracy of the first-order approximation over a large number of revolutions and allows to meet the boundary conditions very accurately.

Analysis

The spacecraft is treated as a point mass. It is equipped with a low thrust constant acceleration engine. All further perturbations are neglected. The regularized equations of motion for the planar ascent can be written as

$$r' = u \tag{1a}$$

$$u' = \mu + 2hr + \varepsilon r^2 e_r \tag{1b}$$

$$v' = \varepsilon r^2 e_\phi \tag{1c}$$

$$h' = \varepsilon(ue_r + ve_\phi) \tag{1d}$$

$$\phi' = v/r \tag{1e}$$

$$t' = r \tag{1f}$$

where r is the radius, ϕ the central angle, u and v are the components of the velocity in radial and circumferential direction, t is the time, and ε the constant small acceleration. The total energy h is used as an additional state variable. The components e_r and e_ϕ of the control $\bar{e}$ stand for

$$e_r = \cos\theta = \cot\theta(1+\cot^2\theta)^{-1/2} \tag{2a}$$

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